The starting assumption made for what follows is that the model should not be analysed from a continuous point of view but rather under a discrete one using empirical methods. Also, an other thing that must be taken into account is that, due to how construction works in game, there are some issues that are quite hard to "put into mathematics". The most obvious of these are

1. No more than 15 CIVs can build another CIV at the same time.

2. Not all work done produces other output.

The easiest way to address these main issues is to not take into account any other aspects of construction apart from a CIV's daily base output of 5. At this point the following code, if run, returns the amount of factories at the end of each month from January 1936 to August 1939. I choose this time period of 44 months since, around mid August 1939 war usually starts and, even if it was convenient to build only CIVs, starting from September 1939 the amount of uncertainty is just to big to be ignored. Also, to simplify things, I worked with a 360 days year (30 days for each month), so the last time mark should correspond to mid August 1939.

The best idea I came up with, was to use following C code (if it looks a bit messy it's because I had to make the comments fit in the page)

```
#include <stdio.h>

int main()
{
    int initial_factories, final_factories, day, month, year, work_done[1000];
    int dedicated_factories[1000], factories_not_assigned, building_slot;

    printf("Insert starting factories: ");
    scanf("%d", &initial_factories);

    final_factories = initial_factories;

    for(building_slot = 0; building_slot < 1000; building_slot++){     /*setting all works done to 0*/
        work_done[building_slot] = 0;
        dedicated_factories[building_slot] = 0;
    }

    for(day = 1; day <= 1320; day++){                                  /*iterating until war starts*/
        building_slot = 0;
        factories_not_assigned = final_factories;

        while(factories_not_assigned > 0){      /*assigning factories to elements in building queue*/
            if(factories_not_assigned >= 15){
                dedicated_factories[building_slot] = 15;
                factories_not_assigned = factories_not_assigned - 15;
            }
            else{
                dedicated_factories[building_slot] = factories_not_assigned;
                factories_not_assigned = 0;
            }
            building_slot ++;
        }

        for(building_slot = 0; building_slot < 1000; building_slot++){/*factories doing work in each slot*/
            if(dedicated_factories[building_slot] > 0){
                work_done[building_slot] = work_done[building_slot] + dedicated_factories[building_slot]*5;

                if(work_done[building_slot] >= 10800){
```

```
                final_factories ++;
                work_done[building_slot] = work_done[building_slot] - 10800;
            }
        }
    }

    if(day % 30 == 0){        /*printing out time marks and final factories*/
        month = day / 30;
        if(month == 1)
            printf("Year 1936\n");
        if(month == 13)
            printf("Year 1937\n");
        if(month == 25)
            printf("Year 1938\n");
        if(month == 37)
            printf("Year 1939\n");

        printf("Month %d, factories count: %d\n", month, final_factories);
    }

  }

  return 0;
}
```

Note: this code does not take into account anything but the simple base daily output of a CIV. However it should not be hard to adjust it in order to make it take into account also other aspects like economic laws, stability, infrastructure and trade (this might actually be a little bit tricky).

After running this a couple times I came up with the following:

1. If you compare the amount of CIVs you have at the end with those you had at the beginning you obtain the growth ratio over the 44 month period considered. Let C be amount of starting CIVs, than we can call this ratio $\gamma(C)$ since it depends on the number of starting CIVs. Now, running the code for growing C gives the empirical result

$$\lim_{C \to +\infty} \gamma(C) \approx 1.781 \tag{1}$$

although the convergence seems to happen quite slowly.

2. I assumed that the progression of CIVs over time is geometric (the discrete equivalent of an exponential growth). This means that if the starting number of CIVs (being the scale factor of the progression) is C_0, than the amount of CIVs you have after n months is

$$C(n) = C_0(r)^n \tag{2}$$

where r is the common ratio. Thus, since we have $r^{44} \approx 1.781$ we obtain $r \approx 1.013$, which is the monthly growth ratio. This means that the amount of CIVs grows roughly about 1.3% each month.

3. If we now assume that the model is $C(t) = C_0(1.013)^t$ we get a progression whose values have a relative error, with respect to the actual values, that is not higher than 6% and decreases over time. This result has to be interpreted in the following way: if you start with C_0 CIVs in January 1936, all building other CIVs, even if C_0 is very high, you won't get any new CIV's until about 4 months have passed because at maximum 15 CIVs can build one new CIV. After this initial period you have a "jump" in your CIV count. So now the question is, what happens next? Well, since all but the last CIV in the construction queue have surely 15 CIVs that are building them, these will progress evenly (remember, I am not taking into account infrastructure) while the last CIV will have a variable amount of progress, which is different from the others. However, once the first CIVs are built, those still under construction progress in the queue and the first of these will, of course, have a different

progress from the others[1]. After a long enough time span, these progresses should be "mixed" enough to allow us to study the CIV growth over time with a continuous model in the sense that the time intervals between a CIV and an other being built should become smaller and smaller.

In conclusion, the examination of this simple case was time consuming and I don't think that I can afford to invest the time needed to conclude it since having to study for university is keeping me quite busy. This being said, I think that an empirical approach is the way to go and that the code I shared above can be easily improved to take into account other aspects of construction like infrastructure, stability, economic policies, etc.

[1] I would hypothesize that, if we call X_s the random variable that gives the construction progress of the slot s, than $X \sim U(0,1)$ (X is uniformly distributed in the interval $[0,1]$). If all the X_t corresponding to slots in which factories are working are independent and identically distributed (i.i.d.), than their sum would have a normal distribution whose mean is the sum of all means μ_t of the X_t's that are being summed, justifying the assumption that on average the construction process is $\frac{1}{2}$. (By the way, to obtain this result X_t must not necessarily have uniform distribution. In fact any distribution would do, as long as the X_t's are i.i.d. as stated by central limit theorem).