Civilian Factory Payout

Personwithhat

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1 Introduction

The following attempts to address the omnipresent decision:
"When should I stop building Civilian Factories, and switch to Military Factories?"

There is a substantial of misinformation, generally spread by hearsay and substantiated by mere opinion. Unfortunately, in all but the most extreme cases, building a Civ does not pay for itself. The truly optimal decision should be to exclusively build Mils.

2 Assumptions and Definitions

Definition:
$d =$ Days since start of construction
$G =$ Daily output of Military factory
$M =$ Days taken to construct a Military Factory, at $d = 0$
$C =$ Days taken to construct a Civilian Factory, at $d = 0$
$K =$ Days taken to construct a Military Factory, at $d > C$
$B =$ Bonus to Military factory output, at $d > C$

1) To simplify the model, we discount production efficiency. We also assume that a Civ builds Mils in a smooth manner and produces a consistent output. E.g. A tenth of a Mil will produce a tenth of a Mil's output.
2) Resource limitations can be safely ignored as they have no direct effect, and are typically resolved through alternative means such as infrastructure/focuses.
3) There is no need to consider consumer goods for the initial case builds, as the number of builder Civs (T) is consistent. In favor of a more "Optimistic" breakpoint, we can ignore consumer good's effect on Y_c
4) Economy laws and Construction/output bonuses are consistent, and may only change once at $d = C$.

With these in mind, we can model the the total military output after d days as:

$$Y = \int_0^d \frac{BGx}{M}\, dx = \frac{BG}{M}\int_0^d x\, dx = \frac{BGd^2}{2M}$$

Assuming construction uses a constant number of Civs that are exclusively building Mils.

3 Case Study

At any point in time there are only two cases to consider. We can construct one Civ that will continue to build Mils in a smooth manner, or we can construct $\frac{C}{M}$ Mils instead.

In Case 1, it takes C days before our new Civ is ready to construct Mils.
The output of our Civ at d days is:

$$Y_c = \int_C^d \frac{BGx}{K}\, dx = \frac{BG(d-C)^2}{2K}$$

In Case 2, we would have constructed Mils instead.
The output of our $\frac{C}{M}$ Mils:

Y_m = Output during construction + Output from C to d days

$$Y_m = \int_0^C \frac{Gx}{M}\, dx + BG(\frac{C}{M})(d-C)$$

$$Y_m = \frac{GC^2}{2M} + BG(\frac{C}{M})(d-C)$$

The break even point is defined as $Y_c = Y_m$. Now we just need to find the appropriate d that satisfies the conditional.

$$Y_c - Y_m = 0$$

$$\frac{BG(d-C)^2}{2K} - \frac{GC^2}{2M} - BG(\frac{C}{M})(d-C) = 0$$

Simplify constants:

$$\frac{G(d-C)^2}{K} - \frac{GC^2}{BM} - 2G(\frac{C}{M})(d-C) = 0$$

$$\frac{GM(d-C)^2}{K} - \frac{GC^2}{B} - 2GC(d-C) = 0$$
$$\frac{M(d-C)^2}{K} - \frac{C^2}{B} - 2C(d-C) = 0$$

Open parenthesis:

$$\frac{M}{K}(d^2 - 2Cd + C^2) - \frac{C^2}{B} - 2Cd + 2C^2 = 0$$

$$\frac{M}{K}d^2 - \frac{M}{K}2Cd + \frac{M}{K}C^2 - \frac{C^2}{B} - 2Cd + 2C^2 = 0$$

Simplify:

$$\frac{M}{K}d^2 - (2 + \frac{2M}{K})Cd + \frac{M}{K}C^2 - \frac{C^2}{B} + 2C^2 = 0$$
$$\frac{M}{K}d^2 - (2 + \frac{2M}{K})Cd + (2 + \frac{M}{K} - \frac{1}{B})C^2 = 0$$

Substitute for the constant ratio, $R = \frac{M}{K}$:

$$Rd^2 - (2 + 2R)Cd + (2 + R - \frac{1}{B})C^2 = 0$$

Solve the quadratic:

$$a = R$$
$$b = -(2 + 2R)C$$
$$c = (2 + R - \frac{1}{B})C^2$$
$$\frac{(2+2R)C \pm \sqrt{((2+2R)C)^2 - 4R(2 + R - \frac{1}{B})C^2}}{2R}$$

Take out 2:

$$\frac{(2+2R)C \pm \sqrt{4(1+R)^2C^2 - 4R(2 + R - \frac{1}{B})C^2}}{2R}$$

$$\frac{(2+2R)C \pm 2\sqrt{(1+R)^2C^2 - R(2 + R - \frac{1}{B})C^2}}{2R}$$

Take out C:

$$\frac{(1+R)C \pm \sqrt{((1+R)^2 - R(2 + R - \frac{1}{B}))C^2}}{R}$$
$$\frac{(1+R)C \pm C\sqrt{(1+R)^2 - R(2 + R - \frac{1}{B})}}{R}$$

Expand:

$$\frac{(1+R)C \pm C\sqrt{R^2+2R+1-2R-R^2+\frac{R}{B}}}{R}$$

Simplify for solution. We can safely ignore $-$ for our model as there is no valid break point for which $d < C$:

$$d = \frac{(1+R)C + C\sqrt{1+\frac{R}{B}}}{R}$$

4 Graphing solution

T = Number of Civs used in initial build
I = Infrastructure used for initial Mil or Civ build
M_b = Initial Mil construction speed bonus
C_b = Initial Civ construction speed bonus
K_b = Mil construction speed bonus, at $d > C$
Q = Infrastructure used for Mil builds, at $d > C$
B = Bonus to Mil output, at $d > C$

Expanding our construction speed for one Mil or Civ:

$$M = \frac{7200}{CivOutput * T * I * M_b} = \frac{7200}{5 * T * I * M_b} = \frac{1440}{TIM_b}$$

$$C = \frac{10800}{CivOutput * T * I * C_b} = \frac{10800}{5 * T * I * C_b} = \frac{2160}{TIC_b}$$

$$K = \frac{7200}{CivOutput * Q * K_b} = \frac{7200}{5 * Q * K_b} = \frac{1440}{QK_b}$$

Substitute for the ratio R:

$$R = \frac{M}{K}$$

$$R = \frac{\frac{1440}{TIM_b}}{\frac{1440}{QK_b}} = \frac{QK_b}{TIM_b}$$

Resolution:

$$d = \frac{(1+R)C + C\sqrt{1+\frac{R}{B}}}{R}$$

$$d = \frac{(1+\frac{QK_b}{TIM_b})C + C\sqrt{1+\frac{\frac{QK_b}{TIM_b}}{B}}}{\frac{QK_b}{TIM_b}}$$

$$d = \frac{(1 + \frac{QK_b}{TIM_b})C + C\sqrt{1 + \frac{QK_b}{BTIM_b}}}{\frac{QK_b}{TIM_b}}$$

$$d = \frac{(1 + \frac{QK_b}{TIM_b})(\frac{2160}{TIC_b}) + (\frac{2160}{TIC_b})\sqrt{1 + \frac{QK_b}{BTIM_b}}}{\frac{QK_b}{TIM_b}}$$

To find how many years it takes after a Civ is built, before we hit the break even point:

$$Y = \frac{d - C}{365}$$

$$Y = \frac{\frac{(1+\frac{QK_b}{TIM_b})(\frac{2160}{TIC_b})+(\frac{2160}{TIC_b})\sqrt{1+\frac{QK_b}{BTIM_b}}}{\frac{QK_b}{TIM_b}} - \frac{2160}{TIC_b}}{365}$$

You can see the graph here, where:

Y = How many years elapse after Civ is built, before it pays itself off.

$x = K_b$